26th Central European Number Theory Conference 2025

Book of Abstracts

University of Salzburg

8th – 12th September 2025

Contents

General Information

Welcome

The Department of Mathematics and the local organizers welcome you to CENT2025 in Salzburg.
In this book you can find some general information, the program, and all abstracts. For additional information and updates visit the conference webpage at `https://math.sbg.ac.at/cent2025/Cent2025.html`.
We hope that you will have a very fruitful and enjoyable time in Salzburg.

The Local Organizers
Salzburg, September 2025

Conference Location

The conference takes place in the Blue Lecture Hall (Blauer Hörsaal, HS 402,H34EG0.F-011) at the “Freisaal”-building (formerly “NaWi”-building) of the University of Salzburg. The address is:

Hellbrunner Str. 34
5020 Salzburg
Austria

The Department of Mathematics can be contacted by phone at +43 662 80445300.

Registration and Opening

Registration of participants takes place on Monday, 8th September 2025, 08:15 - 08:45 in the entrance hall of the conference venue. The conference will be opened at 08:45 in the Audimax.

Conference Dinner

The Conference Dinner takes place on Wednesday (10th September) evening. It’s located at the restaurant ”Goldene Kugel” (Judengasse 3, 5020 Salzburg) and starts at 19:00.
In case you need a special diet, please let us know at the registration.

City Tour

There are no talks scheduled on Wednesday afternoon. Instead you have the possibility to join a tour in the old town of Salzburg. There is a collective walk to the starting point from the university, starting at 14:30. In case you want to get there on your own: The meeting point is at the monument of "Wolfgang Amadeus Mozart" at Mozart Square (Mozartpl. 1, 5020 Salzburg), tour start is at 15:00.

WiFi

There is free WiFi available throughout the conference.
Please log into the eduroam Network with the following credentials:

Username: v1117387@sbg.ac.at
Password: Cent2025math

Scientific Committee

The plenary speakers have been selected by the Scientific Committee consisting of the following professors:

- Attila Bérczes, University of Debrecen
- Andrej Dujella, University of Zagreb
- Ladislav Mišík, University of Ostrava
- Karol Nemoga, Slovak Academy of Sciences
- Attila Pethő, University of Debrecen
- Štefan Porubský, Czech Academy of Sciences
- László Szalay, University of Sopron
- Robert Tichy, Graz University of Technology
- János Tóth, J. Selye University
- Maciej Ulas, Jagiellonian University in Kraków

Local Organizers

The organizers of the conference are:

- Clemens Fuchs
- István Pink
- Carina Premstaller
- Volker Ziegler

Program

Monday, 8th September 2025			
08:15 - 08:45	Registration		
08:45 - 09:00	Opening	Clemens Fuchs	
09:00 - 10:00	Plenary Talk	Kálmán Győry	Chair: Clemens Fuchs
10:00 - 10:30	Coffee Break		
10:30 - 10:55	Contributed Talk	Ingrid Vukusic	Chair: László Remete
10:55 - 11:20	Contributed Talk	Ondřej Turek	
11:20 - 11:35	15 min Break		
11:35 - 12:00	Contributed Talk	Ivan Soldo	
12:00 - 12:25	Contributed Talk	Pascal Jelinek	
12:30 - 14:00	Lunch		
14:00 - 15:00	Plenary Talk	Edita Pelantová	Chair: István Pink
15:00 - 15:45	Coffee Break		
15:45 - 16:10	Contributed Talk	Magdaléna Tinková	Chair: Radan Kučera
16:10 - 16:35	Contributed Talk	Simona Fryšová	
16:35 - 16:50	15 min Break		
16:50 - 17:15	Contributed Talk	István Pink	
17:15 - 17:40	Contributed Talk	András Bazsó	
17:40 - 18:05	Contributed Talk	Sarthak Gupta	

Tuesday, 9th September 2025

09:00 - 10:00	Plenary Talk	Florian Luca	Chair: Maciej Ulas
10:00 - 10:45	Coffee Break & Photo		
10:45 - 11:10	Contributed Talk	Lászlo Tóth	Chair: Magdaléna Tinková
11:10 - 11:35	Contributed Talk	Manfred Madritsch	
11:35 - 11:55	Contributed Talk	Laszlo Remete	
12:00 - 14:00	Lunch		
14:00 - 15:00	Plenary Talk	Lajos Hajdu	Chair: Volker Ziegler
15:00 - 15:45	Coffee Break		
15:45 - 16:10	Contributed Talk	Ana Jurasić	Chair: Ingrid Vukusic
16:10 - 16:35	Contributed Talk	Alan Filipin	
16:35 - 16:50	15 min Break		
16:50 - 17:15	Contributed Talk	Armand Noubissie	
17:15 - 17:40	Contributed Talk	Jitu Leta	

Wednesday, 10th September 2025

09:00 - 10:00	Plenary Talk	Daniel Smertnig	Chair: Edita Pelantová
10:00 - 10:30	Coffee Break		
10:30 - 10:55	Contributed Talk	Roma Kačinskaitė	Chair: László Tóth
10:55 - 11:20	Contributed Talk	Elchin Hasanalizade	
11:20 - 11:35	15 min Break		
11:35 - 12:00	Contributed Talk	Maciej Ulas	
12:00 - 12:25	Contributed Talk	László Szalay	
12:30 - 14:30	Lunch		
14:30 - 15:00	Walk to the city		
15:00 - 17:30	City Tour		
19:00	Conference Dinner		

Thursday, 11th September 2025

09:00 - 10:00	Plenary Talk	Mihai Cipu	Chair: László Szalay
10:00 - 10:45	Coffee Break		
10:30 - 10:55	Contributed Talk	Radan Kučera	Chair: Alan Filipin
10:55 - 11:20	Contributed Talk	Adéla Václavová	
11:20 - 11:35	15 min Break		
11:35 - 12:00	Contributed Talk	Jan Vondruška	
12:00 - 12:25	Contributed Talk	Krystian Gajdzica	
12:30 - 14:00	Lunch		
14:00 - 15:00	Plenary Talk	Jerzy Kaczorowski	Chair: Lajos Hajdu
15:00 - 15:45	Coffee Break		
15:45 - 16:10	Contributed Talk	Joachim König	Chair: Manfred Madritsch
16:10 - 16:35	Contributed Talk	Pavel Francírek	
16:35 - 16:50	15 min Break		
16:50 - 17:15	Contributed Talk	Piotr Miska	
17:15 - 17:40	Contributed Talk	Marian Genčev	
17:40 - 18:05	Contributed Talk	Gergő Batta	

Friday, 12th September 2025

09:00 - 10:00	Plenary Talk	Martin Widmer	Chair: Attila Bérczes
10:00 - 10:30	Coffee Break		
10:30 - 10:55	Contributed Talk	Bartosz Sobolewski	Chair: András Bazsó
10:55 - 11:20	Contributed Talk	Om Prakash	
11:20 - 11:35	15 min Break		
11:35 - 12:00	Contributed Talk	Lorenzo Sauras-Altuzarra	
12:00 - 12:25	Contributed Talk	Mikuláš Zindulka	
12:25 - 12:30	Closing		

Plenary Talks

Mihai Cipu

$D(\pm 1)$-sets in imaginary quadratic fields: old and new

Abstract: A problem of major interest in the study of $D(n)$-tuples is how large their cardinality can be. We consider this question for $D(\pm 1)$-sets consisting of algebraic integers in imaginary quadratic fields. In comparison with the classical set-up, new phenomena appear and a variety of techniques are employed to overcome the ensuing difficulties.
The talk is based on on-going joint work with Yasutsugu Fujita (Nihon University).

Lajos Hajdu

The PTE problem, decomposability of polynomials and Diophantine equations

Abstract: The Prouhet-Tarry-Escott problem, shortly PTE, asks to describe disjoint pairs A and B of sets of integers such that their first k power sum symmetric polynomials are equal for some k. In spite of that only a little is known about such PTE-pairs of sets A, B, the problem has a vast literature.
Another classical problem is related to equations of the type

$$f(x) = g(y) \tag{1}$$

in integers x, y, where f, g are polynomials with rational coefficients, in many cases coming from some 'interesting' family. This problem has been considered by many eminent mathematicians, who obtained several important and deep results. For example, if $f(x) = x(x+1)\dots(x+n-1)$ $(n \geq 2)$ and $g(y) = y^\ell$ $(\ell \geq 2)$, then by a celebrated result of Erdős and Selfridge we know that (1) has no solutions in positive integers x, y. In the general case one may try to give finiteness results for the solutions of (1) by a deep theorem of Bilu and Tichy. In this the decomposability properties of f and g plays a vital role.
In the talk we provide a link between the PTE problem and decomposability of polynomials, so by the Bilu-Tichy theorem connecting the three subjects mentioned in the title. In particular, first we study the structure of partitions of $A \subseteq \{1, 2, \dots, n\}$ in k-sets such that the first $k-1$ symmetric polynomials

of the elements of the k-sets coincide. Then using the above mentioned link we apply this result to derive a decomposability theorem for the polynomial $f_A(x) := \prod_{x\in A}(x-a)$. After that, extending several results from the literature, by the Bilu-Tichy theorem we prove finiteness theorems for the integer solutions x, y of the Diophantine equation $f_A(x) = g(y)$ where $g(y) \in \mathbb{Q}[y]$ and on shifted power values of $f_A(x)$. Finally, extending our approach, we study (1) in the case where the roots of at least one of f, g are simple and rational. In this generality we are only able to get information concerning the degrees of f and g - however, by examples we show that all the possible cases can occur, indeed. During the talk, we shall focus on the description of structures, principles, connections and provide several examples, and try to avoid technical details.
The new results presented are joint with Á. Papp and R. Tijdeman.

Kálmán Győry

Effective reduction theory of integral polynomials of given discriminant and related topics

(joint survey with Jan-Hendrik Evertse)

Abstract: We give a survey on the effective reduction theory of integral polynomials of given discriminant and its applications. For polynomials in $\mathbb{Z}[X]$, the classical $\mathbb{Z}$-equivalence and $GL(2,\mathbb{Z})$-equivalence preserve the degree and discriminant as invariants. The effective reduction theory we consider asks to find, for a given polynomial f, a $\mathbb{Z}$-equivalent or $GL(2,\mathbb{Z})$-equivalent integral polynomial whose coefficients are effectively bounded above in terms of the degree and discriminant of f.
We discuss the classical effective results of this type of Lagrange (1773) and Hermite (1851) on quadratic and cubic polynomials, the general ineffective result of Birch and Merriman (1972) and the general effective finiteness theorems of Győry (1973), obtained independently, and of Evertse and Győry (1991) who settled the above mentioned problem in full generality, using Győry's effective finiteness results on unit equations. Later Evertse, Győry and others obtained several generalizations and applications; see Evertse and Győry, Discriminant Equations in Diophantine Number Theory, Cambridge, 2017. Together with Bhargava, Evertse, Remete and Swaminathan, in 2023 we compared our general effective theorems mentioned above with a long-forgotten result of Hermite (1857), who used another notion of equivalence, and we pointed out that this is weaker than $\mathbb{Z}$-equivalence and $GL(2,\mathbb{Z})$-equivalence. This and many other recent results inspired us with Evertse to write a longer joint survey paper on the subject which will appear soon. In

our talk we give a brief overview of the most important consequences, generalizations and various applications of the effective reduction theory of integral polynomials, e.g. for monogenic number fields , monogenic and rationally monogenic orders, and so on. Some related topics will also be mentioned.

Jerzy Kaczorowski

Twists of L-functions and their applications

Abstract: The talk will present recent developments in the theory of twists of L-functions, a subject we have been pursuing in a series of joint works with Alberto Perelli over the past two decades. Twists provide a powerful and indispensable tool for establishing converse theorems, which characterize classical L-functions through their analytic properties. A natural framework for these investigations is offered by the Selberg class and its extended version, whose structural description may itself be viewed as a broad converse theorem. Two central conjectures concern these classes: the degree conjecture, asserting that every element of the extended Selberg class has a nonnegative integral degree, and the conjecture that elements of the Selberg class of integral degree arise from automorphic representations of arithmetic groups. Proving these conjectures is expected to be extremely challenging with the methods currently available. In this lecture, I will outline the key ideas showing how the theory of twists can be used to confirm these conjectures in the case of low-degree L-functions.

Florian Luca

On the distance between factorials and repunits

Abstract: We show that if $n \geq n_0$, $b \geq 2$ are integers, $p \geq 7$ is prime and

$$n! - (b^p - 1)/(b - 1) \geq 0,$$

then $n! - (b^p - 1)/(b - 1) \geq 0.5 \log\log n / \log\log\log n$. Further results are obtained, in particular for the case $n! - (b^p - 1)/(b - 1) < 0$. This research was inspired by the Brocard–Ramanujan problem which asks for all the positive integer solutions (n, b) of the Diophantine equation $n! + 1 = b^2$.
This research is supported by the 2024 ERC Synergy Project DynAMiCs and is joint with Michael Filaseta.

Edita Pelantová

Cantor real numeration system

Abstract: We consider a numeration system which is a common generalization of the positional systems introduced by Cantor and Rényi. In the Cantor real system, number representations are obtained using a composition of β_k-transformations for a given sequence of real bases $\mathcal{B} = (\beta_k)_{k\in\mathbb{Z}}$, $\beta_k > 1$. We present a quick survey on classical results on Rényi numeration systems from the combinatorial, algebraic and arithmetical point of view. Then we discuss which properties of the Rényi system and in what form remain preserved in the generalized system and what new phenomena the generalized system brings. This study has been led in collaboration with Zuzana Masáková, Savinien Kreczman, Célia Cisternino and Emilie Charlier.

Daniel Smertnig

Applications of unit equations to lineargroups and weighted automata

Abstract: I will present recent applications of unit equations to finitely generated matrix groups. In particular, we obtain arithmetic characterizations of group representations that are

1. epimorphic images of monomial representations;
2. epimorphic images of block-triangular representations with monomial diagonal blocks.

These results can be seen as an extension of the classical Burnside-Schur Theorem, which states that finitely generated torsion matrix groups are finite. Our work is motivated by applications to weighted automata over fields (which are noncommutative multivariate generalizations of linear recurrence sequences). For weighted automata, our results allow us to partially prove a natural ambiguity hierarchy and its decidability.
The talk is based on joint work with Antoni Puch at the University of Warsaw and on joint work with Jason Bell.

Martin Widmer

Universal quadratic forms over infinite extensions

Abstract: By Lagrange's Theorem the sum of four squares is a universal quadratic form, i.e., represents all (totally) positive integers. This is almost never the case when we replace the rational integers by the integers of a totally real number field. However, there is always some universal quadratic form. The situation is fundamentally different for infinite totally real extensions. Daans, Kala and Hang Man have shown that in this case the Northcott property is an obstruction to the existence of a universal quadratic form, and asked whether it is the only obstruction. We show that most (in a suitable topological sense) totally real extensions do not have a universal quadratic form or the Northcott property. The main tool is a new obstruction to the existence of a universal quadratic form. I will discuss the result and some ideas of the proof.This is joint work with Nicolas Daans, Vitezslav Kala, Siu Hang Man, and Pavlo Yatsyna.

Contributed Talks

Gergő Batta

On Diophantine graphs

Abstract: A set of n distinct positive integers is called a Diophantine n-tuple, if the product of any two distinct terms from the set increased by one is a square. In the present talk, extending the problem of Diophantine tuples, we study Diophantine graphs. Given a finite set V of positive integers, the induced Diophantine graph $D(V)$ has vertex set V, and two numbers in V are linked by an edge if and only if they form a Diophantine pair. A finite graph G is a Diophantine graph if it is isomorphic to $D(V)$ for some V. We present various results for Diophantine graphs, concerning representability and extendability questions, related to the edge density, and also for their chromatic number.

András Bazsó

Singmaster-type results for Stirling numbers and some related diophantine equations

Abstract: In the talk, motivated by the work of David Singmaster on binomial coefficients, we study the number of times an integer can appear among the Stirling numbers of both kinds. We provide an upper bound for the occurrences of all the positive integers, and present some numerical results and conjectures concerning the related diohantine equations. The presented results are joint with István Mező, Ákos Pintér and Szabolcs Tengely.

Alan Filipin

On the Concatenations of two repdigits in some integer sequences

Abstract: Let $b \geq 2$ be a positive integer. In this talk we prove that for a fixed b, there exist only finitely many Pell and Pell-Lucas numbers that can be written as concatenations of two repdigits in base b. In the proof we use standard methods like the application of Baker's theory on linear forms in logarithms of algebraic numbers, the Baker-Davenport reduction procedure (version of Dujella and Pethő) and lot of computations which were done with the help of a computer programs in Maple. As illustration, we solve the

problem completely for $2 \leq b \leq 10$. This is joint work with K. N. Adédji, S. E. Rihane and A Togbé.

Pavel Francírek

On Sinnott's index formula

Abstract: Sinnott's formula expresses the index of the Stickelberger ideal for any imaginary abelian number field M as the product of the relative class number of M, the index $(e^- R : e^- U)$, and the inverse of the Hasse index (which equals 1 or 2). The Sinnott module U is a $\mathbb{Z}[G]$-submodule of the group ring $\mathbb{Q}[G]$, where $G = \mathrm{Gal}(M/\mathbb{Q})$, but the structure of U is not determined solely by the Galois group G. Instead, it depends on the inertia subgroups of the ramified primes and their Frobenius automorphisms. The motivation for this work is to provide upper bounds on the index $(e^- R : e^- U)$ for all imaginary abelian fields M whose Galois group $\mathrm{Gal}(M/\mathbb{Q})$ is isomorphic to a fixed finite abelian group G. We derive an explicit upper bound for this index that depends only on the group G and the specified embedding of complex conjugation in G.

Simona Fryšová

Non-decomposable quadratic forms over biquadratic fields and the simplest cubic fields

Abstract: Non-decomposable quadratic forms over integers have been studied for years. They become especially interesting if the coefficients are taken to be algebraic integers from a number field. Our work is focused on totally real biquadratic fields and the simplest cubic fields. The main result of our paper for biquadratic fields is that, for every totally real biquadratic field, there exists a non-decomposable binary quadratic form. We obtain the same result also for the simplest cubic fields. Moreover, for these fields, we give a lower bound for the number of all non-decomposable binary quadratic forms up to equivalence. This is joint work with Magdaléna Tinková.

Krystian Gajdzica

Log-behavior of infinite product generating functions

Abstract: In 2015, DeSalvo and Pak reproved Nicolas' theorem stating that the partition function $p(n)$ is log-concave for $n > 25$. Their paper was the

initial point of the broad research concerning similar phenomena for other partition statistics. In particular, the log-concavity and the log-convexity properties enjoy great popularity these days. In this talk, we provide a quite new perspective on these issues.
Let $E \subseteq \mathbb{N}_{\geq 2}$ be fixed. We consider a family $\{p_{\ell,E}(n)\}_{\ell,n}$ of double sequences defined via the generating function of the form:

$$\sum_{n=0}^{\infty} p_{\ell,E}(n)\, q^n := \prod_{n=1, n\notin E}^{\infty} (1-q^n)^{-f_\ell(n)},$$

where $\{f_\ell\}_\ell$ are functions on $\mathbb{N} \setminus E$ satisfying certain growth conditions. In such a setting, we fix n, and investigate the log-behavior of $p_{\ell,E}(n)$ as ℓ grows. It turns out that there is an integer n_E such that for all $a, b \geq n_E$ the log-behaviors of $p_{\ell,E}(a)$ and $p_{\ell,E}(b)$ are the same for all sufficiently large values of ℓ whenever $a \equiv b \pmod{r_E}$, where r_E is an integer that depends essentially on the set E.

Marian Genčev

Polylogarithm Relations and Zeta-Star Values via Binomial-Harmonic Sums

Abstract: We investigate a class of binomial-harmonic sums of the form

$$M_n^{(s_1,\ldots,s_d)}(a,p) := \sum_{k=1}^{n} \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \cdot \zeta_k^\star(s_1,\ldots,s_d;a),$$

where $s_i \in \mathbb{N}$, $a, p \in \mathbb{R}$, and

$$\zeta_k^\star(s_1,\ldots,s_d;a) := \sum_{k \geq n_1 \geq \cdots \geq n_d \geq 1} \frac{a^{n_d}}{n_1^{s_1} \cdots n_d^{s_d}}$$

denotes the generalized multiple harmonic-star value. These values serve as a tool to establish transformation identities linking $M_n^{(s_1,\ldots,s_d)}(a,p)$ to multiple polylogarithms. The resulting identities yield new reduction formulas lowering the depth of certain polylogarithms and provide novel representations of multiple zeta-star values via arithmetic means of the generalized harmonic-star numbers. The analytic approach relies on summation transformations and the Toeplitz limit theorem.

Sarthak Gupta

On a generalization of a problem of Erdős-Selfridge

Abstract: The Erdős-Selfridge theorem (1975) says that the product of consecutive positive integers is never a perfect power, i.e.

$$x(x+1)(x+2)\cdots(x+k-1)=y^n$$

has no solutions in positive integers x, k, y, n with $k \geq 2$ and $n \geq 2$.
Several generalizations and extensions of this problem have been studied by Saradha, Shorey (2001, 2003, 2007), Kulkarni and Sury (2003), Bennett and Siksek (2017), Győry (1997), Győry, Hajdu and Pintér (2009).
In our paper, we consider the equation

$$x(x+1)(x+2)\cdots(x+k-1)(x+a_1)\cdots(x+a_t)=g(y)$$

in integers x, y where $k \geq 2$ and $a_1, a_2, \cdots, a_t$ are fixed integers and $g(y) = y^n, ay^n + b$ $(n \geq 2)$ or g is an arbitrary polynomial. We present finiteness statements using Schinzel-Tijdeman theorem (1976), Brindza's result (1984) and the Bilu-Tichy theorem (2000).

Elchin Hasanalizade

Some results on the unitary analogue of the Lehmer totient problem

Abstract: A composite positive integer n with more than one prime divisor is called *Subbarao* if $\phi^*(n)$ divides $n-1$, where $\phi^*(n)$ is the unitary analogue of the Euler's totient function. No Subbarao number is known, nor has it been proved that they don't exist. We find the possible forms of Subbarao numbers. We also show that there is no Subbarao number in the balancing sequence $(B_n)_{n\geq 0}$ given by $B_0 = 0$, $B_1 = 1$ and the recursive relation $B_n = 6B_{n-1} - B_{n-2}$ for $n \geq 2$.

Pascal Jelinek

Joint distribution of the sum of digits function in two bases

Abstract: In 2019, La Bretèche, Stoll, Tennenbaum showed that the ratio $s_p(n)/s_q(n)$ of the sum of digits functions in two multiplicatively independent

bases p and q is dense in $\mathbb{Q}^+$. Spiegelhofer proved that when $p = 2$ and $p = 3$, the ratio 1 is attained infinitely many times, which he extended jointly with Drmota to arbitrary values in $\mathbb{Q}^+$. In this talk, I generalize this result further, showing that for two arbitrary multiplicatively independent bases, $s_p(n)/s_q(n)$ attains every value in $\mathbb{Q}^+$ infinitely many times.

Ana Jurasić

On the Triangular Diophantine m-tuples

Abstract: Since Diophantuses discovery of the set $\{\frac{1}{16}, \frac{33}{16}, \frac{17}{4}, \frac{105}{16}\}$ of four positive rational numbers where the product of each two of them increased by 1 is a square, various generalizations have been studied. We study one variation of the problem of Diophantus, which includes triangular numbers, i.e., numbers of the form $\Delta_k = \frac{k(k+1)}{2}$, $k \in \mathbb{N}$. In the talk, our result in this variant of the problem of Diophantus will be presented, together with a small survey of previously obtained results and the work in progress.
For an integer n, a set $\{a_1, \ldots, a_m\}$ of m distinct positive integers with $a_i a_j + n$ a triangular number for $1 \le i < j \le m$ is called a Triangular Diophantine m-tuple with the property $D(n)$ or simply a Triangular $D(n)$-m-tuple. We proved that there does not exist a Triangular $D(3)$-quadruple of the form $\{3, 6, c, d\}$. This is joint work with Alan Filipin and László Szalay.

Roma Kačinskaitė

On classes of zeta-functions and universality

Abstract: The classes of zeta-functions play a significant role in modern analytic number theory as generalizations of the well-known classical Riemann zeta function $\zeta(s)$, $s = \sigma + it$. One of the most remarkable properties of the zeta- and L-functions is the so-called universality property, which says that any analytic function satisfying some natural conditions can be simultaneously approximated by the imaginary shifts of the zeta-function in the right-half plane of the critical strip.
The first result related to the mixed simultaneous approximation by the shifts of a pair consisting of the function $\varphi(s)$ belonging to the rather general class of Matsumoto zeta-functions and the periodic Hurwitz zeta-function $\zeta(s, \alpha; \mathfrak{B})$ was obtained in 2015. In the talk, firstly, we briefly overview joint results obtained by R. Kačinskaitė, K. Matsumoto and Ł. Pańkowski from 2015, and, next, we present recent investigations of discrete type.

The talk by bilateral grant of the Research Council of Lithuania and Japan Society for the Promotion of Science No. S-LJB-25-1 is supported.

Joachim König

Stability questions in arithmetic dynamics

Abstract: A key problem in arithmetic dynamics is the question whether a given polynomial (or rational function) over a number field is "stable" (or "dynamically irreducible"), meaning that all of its iterates are irreducible. This question is relevant both globally and locally. While global stability is expected to be the default case, several previous works have given rise to the expectation that the set of primes modulo which a given polynomial f remains stable (also called "stable primes") is usually a very small set. However, proven results were available only in very special cases. In this talk, I will exhibit and apply a new group-theoretical approach which shows for the first time that, indeed, for "most" polynomials of a given degree (in a concrete sense), the set of stable primes is of density 0.

Radan Kučera

Bases of the Stickelberger ideal of a cyclotomic field

Abstract: For a cyclotomic field $K = \mathbb{Q}(\zeta_m)$, where $\zeta_m = \cos\frac{2\pi}{m} + i\sin\frac{2\pi}{m}$, $m \in \mathbb{Z}$, $m > 2$, the Stickelberger ideal consists of all elements of the integral group ring $\mathbb{Z}[\mathrm{Gal}(K/\mathbb{Q})]$ that can be shown to be annihilators of the class group $\mathrm{Cl}(K)$ of K, using Stickelberger's factorization of the mth power of a Gauss sum and Čebotarev's density theorem.
The Stickelberger ideal of a cyclotomic field is a finitely generated free $\mathbb{Z}$-module, so it has a $\mathbb{Z}$-basis. In this talk, we shall recall the definition and the main properties of the Stickelberger ideal of a cyclotomic field and explain why it is useful to know explicit $\mathbb{Z}$-bases, even though there is no elegant definition of such a basis for a general cyclotomic field.

Jitu Leta

Linear independence of continued fractions with algebraic terms

Abstract: We give conditions on sequences of positive algebraic numbers $\{a_{n,j}\}_{n=1}^{\infty}$, $j = 1, \ldots, M$ and number field $\mathbb{K}$ to ensure that the numbers

defined by the continued fractions $[0; a_{1,j}, a_{2,j}, \ldots]$, $j = 1, \ldots, M$ and 1 are linearly independent over $\mathbb{K}$. This is joint work with Jaroslav Hančl and Mathias Laursen .

Manfred Madritsch

Waring's problem for subpolynomial functions

Abstract: For a given subset $A \subset \mathbb{N}$ of the natural numbers, Waring's problem asks for the existence of a positive integer $s \geq 1$ such that every positive integer $N \geq 1$ has (at least) one representation of the form

$$N = a_1 + \cdots + a_s,$$

where $a_i \in A$ for $1 \leq i \leq s$. The original problem considers the case of the set of k-th powers, *i.e.* $A = \{n^k \colon n \in \mathbb{N}\}$ with $k \geq 2$ an integer.
We denote by B the set of germs at $+\infty$ of continuous real functions on $\mathbb{R}$. Then a Hardy field is any subfield of B, which is closed under differentiation and we denote by $\mathbf{U} \subset B$ the union of all Hardy fields. Finally we call a function $f \in \mathbf{U}$ subpolynomial if there exists $k \in \mathbb{N}$ such that $|f(x)| < x^k$ for all sufficiently large x.
In the present talk we want to consider a variant of Waring's problem with $A = \{\lfloor f(n) \rfloor : n \in \mathbb{N}\}$, where f is a subpolynomial function. The objective is a formula for s depending only on the growth rate of f, similar to the formula for the k-th powers in the original case.

Piotr Miska

On denseness of the quotient sets of sufficiently large subsets of $\mathbb{N}$

Abstract: We say that a set $A \subseteq \mathbb{N}$ is (N)-dense if the set

$$R(A, B) = \left\{ \frac{a}{b} : a \in A, b \in B \right\}$$

is dense in $[0, \infty)$ for each infinite set $B \subseteq \mathbb{N}$. In this talk I will give new characterizations of (N)-denseness of a subset of $\mathbb{N}$. Namely, I shall prove that a set $D \subset \mathbb{N}$ is (N)-dense if and only if for any infinite subsets A, B of $\mathbb{N}$ such that $A \cup B = D$ the quotient set

$$R(A; B) = \left\{ \frac{a}{b} :\, a \in A, b \in B \right\}$$

is dense in the set of non-negative real numbers. Furthermore, I will discuss multi-dimensional generalizations of two results. The first one, by Bukor, Erdős, Šalát, and Tóth, concerns partitions of (N)-dense sets. The latter one, of Bukor and Tóth, gives the relationship between lower and upper asymptotic densities of subsets of $\mathbb{N}$ and the denseness of their ratio sets in $[0, \infty)$.
The talk is based on a joint work with János T. Tóth (J. Selye University in Komárno).

Armand Noubissie

On Growth of Multi-recurrence sequence over function fields

Abstract: The Skolem Problem asks to determine whether a given integer linear recurrence sequence has a zero term. This problem, whose decidability has been open for 90 years, arises across a wide range of topics in computer science and dynamical system. In 1977, A generalization of this problem was made by Loxton and Van der Poorten who conjectured that for any $\epsilon > 0$ and $\{u_n\}$ a linear recurrence sequence with dominant (s) roots > 1 in absolute value, there is a effectively computable constant $C(\epsilon)$, such that if $|u_n| < (\max_i\{|\alpha_i|\})^{n(1-\epsilon)}$, then $n < C(\epsilon)$. Using results of Schmidt and Evertse, a complete non-effective (qualitative) proof of this conjecture was given by Fuchs and Heintze (2021) and, independently, by Karimov and al. (2023). In this talk, we prove a quantitative version of that result by giving an explicit upper bound for the number of solutions. Moreover, we give a function field analogue on growth of multi-recurrence, answering a question posed by Fuchs and Heintze when proving a bound on the growth of linear recurrences in function fields and generalizing a result of Fuchs and Pethő. This is joint work with Clemens Fuchs.

István Pink

On the Diophantine equation $2^s + p^k = m^2$ with a Fermat prime p

Abstract: Ramanujan conjectured that the Diophantine equation

$$2^s - 7 = m^2$$

has five solutions in positive integers, namely $(s, m) = (3, 1), (4, 3), (5, 5), (7, 11)$ and $(15, 181)$. His conjecture was proved by Nagell. The generalized Ramanujan-Nagell equation

$$2^s + D = m^2 \tag{2}$$

in positive integers s and m, where $D \neq 0$ is an integer parameter, was considered by several authors. In this talk, we consider equation (2) when the parameter D is of the form $D = p^k$, where k is a nonnegative integer and $p = 2^{2^\ell} + 1$ is a Fermat prime. Namely, our equation takes the form

$$2^s + p^k = m^2 \qquad \text{with a Fermat prime} \quad p = 2^{2^\ell} + 1. \tag{3}$$

In our talk, we find all the nonnegative integer solutions (m, p, k, s) of (3). This is a joint work with Florian Luca.

Om Prakash

Universality beyond quadratic forms

Abstract: A universal quadratic form is a positive definite quadratic form with integral coefficients that represents all positive integers – a classical example being the sum of four squares $x^2 + y^2 + z^2 + w^2$. The 290-Theorem of Bhargava and Hanke characterizes positive definite quadratic forms over rational integers are universal as exactly the forms that represent $1, 2, 3, \ldots, 290$. In this talk, I will discuss universality of higher degree forms (i.e. homogeneous polynomials of degree $m > 2$), and I will prove that no statement like the 290-Theorem can hold for them. If time permits, I will conclude with the more general case of forms over totally real number fields. This is a joint work with Vítězslav Kala.

Laszlo Remete

Monogenity of Power Compositional Polynomials

Abstract: Several new results on the index of a power composition polynomial of the shape $f(x^k)$, $k \in \mathbb{N}$, have been obtained in recent years. We showed that if $f(x)$ is monogenic and $f(0)$ is squarefree, then the monogenity of $f(x^k)$ depends only on the p-index of $f(x^p)$, where $p \mid k$.
If $f(x)$ splits completely modulo p, then the p-index of $f(x^p)$ can be determined parametrically, thus starting from a monogenic polynomial, it is possible to easily generate infinitely many monogenic polynomials of higher degrees.

On the other hand, if $f(x)$ is irreducible modulo p, then p divides the p-index of $f(x^p)$, if and only if the length of the period of the linear recurrence sequence, with characteristic polynomial $f(x)$, is equal modulo p and modulo p^2. This connects the problem of the monogenity to some other classical porblems in number theory.
We illustrate our method through famous classical examples. This is a joint result with Sumandeep Kaur and Surender Kumar.

Lorenzo Sauras-Altuzarra

Advances in the Closed-Form Calculation of Integer Sequences

Abstract: Drawing on Matiyasevich's framework, Mazzanti (2002) and Marchenkov (2007) achieved a major breakthrough in recursion theory: any "usual" sequence of nonnegative integers (or, more precisely, any univariate Kalmar elementary function) admits a closed-form representation. The considered closed forms, called arithmetic terms, are built inductively from nonnegative integer constants and variables using only the operations of addition, truncated subtraction, multiplication, integer division, and exponentiation. In this presentation, we will explore several representative arithmetic terms and briefly outline the techniques applied to construct them. The results presented here are the product of joint work with Mihai Prunescu and Joseph M. Shunia.

Bartosz Sobolewski

Binary sequences meet the Fibonacci sequence

Abstract: We introduce a new family of number sequences $(f(n))_{n\in\mathbb{N}}$, governed by the recurrence relation

$$f(n) = af(n - u_n - 1) + bf(n - u_n - 2),$$

where $\mathbf{u} = (u_n)_{n\in\mathbb{N}}$ is a sequence with values $0, 1$. Our study focuses on the properties of the sequence of quotients $h(n) = f(n+1)/f(n)$ and its set of values $\mathcal{V}(f) = \{h(n) : n \in \mathbb{N}\}$ for various $\mathbf{u}$. We give a sufficient condition for finiteness of $\mathcal{V}(f)$ and automaticity of $(h(n))_{n\in\mathbb{N}}$, which holds in particular when $\mathbf{u}$ is the famous Prouhet–Thue–Morse sequence. On the other hand, we prove that the set $\mathcal{V}(f)$ is infinite for other special binary sequences $\mathbf{u}$, and obtain a trichotomy in its topological type when $\mathbf{u}$ is eventually periodic. Joint work with Piotr Miska and Maciej Ulas.

Ivan Soldo

The extendibility of the triangular $D(-1)$-pair $\{1, 2\}$

Abstract: The triangular $D(-1)$-tuple is the set of the positive integers with the property that the product of any two of them decreased by 1 is the triangular number. We prove that the only triangular $D(-1)$-triples of the form $\{1, 2, c\}$, $c = 2^n p$, where n is a non-negative integer and p is a prime, are those with $c \in \{11, 46, 352, 11936\}$. Further, we prove that for these c's no triangular $D(-1)$-quadruple of the form $\{1, 2, c, d\}$ exists. The results were obtained based on joint work with Mirela Jukić Bokun.

László Szalay

Linear vector recurrences

Abstract: There is given the vector recurrence

$$\mathbf{v}_n = \mathbf{A}\mathbf{v}_{n-1} \qquad (n \geq 1).$$

Its natural generalization is

$$\mathbf{v}_n = \mathbf{A}_1\mathbf{v}_{n-1} + \cdots + \mathbf{A}_s\mathbf{v}_{n-s} \qquad (n \geq s), \tag{4}$$

where $\mathbf{v}_j$ is a column vector with k components and $\mathbf{A}_i$ is a $k \times k$ complex matrix. For example, if $s = 2$ and $k = 2$, then the system

$$\begin{aligned} x_n &= a_{11}x_{n-1} + a_{21}y_{n-1} + a_{12}x_{n-2} + a_{22}y_{n-2} \\ y_n &= b_{11}x_{n-1} + b_{21}y_{n-1} + b_{12}x_{n-2} + b_{22}y_{n-2} \end{aligned}$$

is equivalent to a particular case of (4). The main question is how to separate the mixed component sequences.
The general results on (4) can play an important role in the solution of several combinatorial and number theoretical problems. As illustration, we analyze some old and new questions. For example, we handle the case of so-called k-periodic linear recurrences.

Magdaléna Tinková

Non-decomposable quadratic forms over totally real number fields

Abstract: Non-decomposable quadratic forms with integer coefficients were studied, for example, by Mordell (1930, 1937) and Erdős and Ko (1938, 1939).

However, we know much less about them if their coefficients belong to the ring of algebraic integers of a totally real number field. Some of our new results are general, but one part is restricted to the case of binary quadratic forms over real quadratic fields. For them, we provide some bounds on the number of such non-decomposable quadratic forms, show that their number is rather large for almost all quadratic fields, or give their whole structure for several examples of these fields. We also show a relation between them and the problem of n-universal quadratic forms. This is joint work with Pavlo Yatsyna.

László Tóth

On the asymptotic density of k-tuples of positive integers with pairwise non-coprime components

Abstract: We use the convolution method for arithmetic functions of several variables to reprove an asymptotic formula concerning the number of k-tuples of positive integers $\leq x$ with certain arbitrary coprimality conditions, obtained by Arias de Reyna and Heyman (2015) using a different approach. Then we deduce an asymptotic formula for the number of k-tuples of positive integers with components which are pairwise non-coprime and $\leq x$. More generally, we obtain similar results for the number of k-tuples $(n_1, \ldots, n_k) \in \mathbb{N}^k$ such that at least r pairs (n_i, n_j), respectively exactly r pairs are coprime. Our results answer the questions raised by Moree (2005, 2014), and generalize and refine related results obtained by Heyman (2014) and Hu (2014).

Ondřej Turek

Continued fractions with even-order partial quotients equal to 1

Abstract: A famous result of M. Hall from 1947 says that every $x \in \mathbb{R}$ can be written as a sum of two continued fractions whose partial quotients do not exceed 4, i.e.,

$$x = [a_0; a_1, a_2, a_3, \ldots] + [b_0; b_1, b_2, b_3, \ldots] \quad \text{with } a_j, b_j \leq 4 \text{ for each } j \in \mathbb{N}.$$

Similarly, every real number $x \geq 1$ is representable as a product of two such continued fractions.

In the talk, we focus on representations of numbers as sums and products of continued fractions with bounded even-order partial quotients. We show that every $x \in \mathbb{R}$ can be written as a sum

$$x = [a_0; a_1, 1, a_3, 1, \ldots] + [b_0; b_1, 1, b_3, 1, \ldots]$$

with $a_{2j} = b_{2j} = 1$ for each $j \in \mathbb{N}$, and every $x > 0$ can be written as a product

$$x = [a_0; a_1, 1, a_3, 1, \ldots] \cdot [b_0; b_1, 1, b_3, 1, \ldots]$$

with $a_{2j} = b_{2j} = 1$ for each $j \in \mathbb{N}$. We also comment on the Hausdorff dimension of the set

$$S = \{[0; a_1, 1, a_3, 1, \ldots] \, ; \, a_{2n-1} \in \mathbb{N} \text{ for all } n \in \mathbb{N}\}$$

consisting of continued fractions whose even-order partial quotients are all equal to 1.

Maciej Ulas

On a class of difference equations involving floor function

Abstract: We present a complete analysis of the long-time behaviour of the recurrence

$$x_n = a\lfloor x_{n-1} \rfloor + b x_{n-1},$$

with $a, b \in \mathbb{R}_{\geq 0}$ and $x_0 \in \mathbb{R}_+$, where a full classification of asymptotic behaviour is obtained in terms of the parameters a, b. We also provide a general formula for the explicit solution of difference equations of the form

$$x_n = a_n \lfloor b_n x_{n-1} \rfloor + c_n x_{n-1} + d_n,$$

where $(a_n)_{n\in\mathbb{N}}, (b_n)_{n\in\mathbb{N}}, (c_n)_{n\in\mathbb{N}}, (d_n)_{n\in\mathbb{N}}$ are integer-valued sequences. This allows us to prove that, under suitable assumptions, such recurrences generate sequences satisfying linear recurrence relations with constant coefficients. Moreover, we analyze in detail the dynamics of the equation $x_n = \lfloor a x_{n-1} \rfloor$, proving several results and formulating open problems. Our experimental findings suggest strong connections with the arithmetic nature of the parameter a, particularly in the case when a is a Pisot number. In particular, we prove that if $a = A + \sqrt{B}, A \in \mathbb{Z}, B \in \mathbb{N}_+$, where B is not a square, satisfies $|A - \sqrt{B}| < 1$, then the sequence $(x_n)_{n\in\mathbb{N}}$, where $x_0 = 1, x_n = \lfloor a x_{n-1} \rfloor$, satisfies linear difference equation with constant coefficients.
The talk is based on a joint work with Bartosz Sobolewski.

Adéla Václavová

Proof of Klaška-Skula Theorem using Class Field Theory

Abstract: In [1] Klaška and Skula proved that for each prime p, all polynomials in the set of all monic cubic polynomials with integer coefficients and a given discriminant D have the same type of factorization modulo p, if $D \in \mathbb{Z}$ is square-free and the class number of $\mathbb{Q}(\sqrt{-3D})$ is not divisible by 3. In their quite extensive proof, they use only elementary methods. However, it is possible to give a more conceptual proof using tools of Class Field Theory and the classical reflection theorem due to Scholz (see for example [3]). Moreover, the result can be generalized using Theorem by Kondo (see [2]): the statement remains valid under the weaker assumption that $D \in \mathbb{Z}$ is the discriminant of some quadratic number field instead of the original condition that $D \in \mathbb{Z}$ is square-free. In addition, we present a similar result concerning type of factorization modulo a prime p of polynomials of degree 4, assuming stronger conditions on $D \in \mathbb{Z}$, more specifically $D \in \mathbb{Z}$ is the discriminant of some quadratic number field, D is divisible by at most two distinct primes and the class number of $\mathbb{Q}(\sqrt{D})$ is not divisible by 3.

References

[1] J. Klaška, L. Skula, *Law of inertia for the factorization of cubic polynomials - the case of discriminants divisible by three*, Mathematica Slovaca, Vol. 66, No. 4 (2016), 1019–1027.

[2] T. Kondo, *Algebraic Number Fields with the Discriminant Equal to That of a Quadratic Number Field*, Journal of the Mathematical Society of Japan, Vol. 47, No. 1 (1995), 31–36.

[3] L. C. Washington, *Introduction to Cyclotomic Fields*, Graduate Texts in Mathematics, Springer, New York, 1997.

Jan Vondruška

Circular units in the compositum of orthogonal cyclic fields with degrees being powers of a fixed odd prime

Abstract: In [1], the author introduces the group of circular units of an abelian field K (a field with an abelian Galois group over the rationals), and calculates its index in the group of all units of K. In [2], the authors look at a more specific case when K is a compositum of some t arithmetically

orthogonal cyclic fields (i.e. fields with cyclic Galois groups over the rationals, which moreover satisfy the condition that whenever any prime ramifies in one of these fields, it splits completely in all the others), each of which is of degree ℓ, where ℓ is an odd prime. They find new units that fall outside of the original group from [1], strengthening the result in this case; finally, they use these new units to obtain a result on the annihilation of the ℓ-Sylow subgroup of the ideal class group of K. However, the class of fields satisfying the above restrictions is quite small, and since the result is rather strong, it begs the question – can it be generalized to a larger class of fields? This talk will focus on a result that does just that. Instead of requiring each of the t cyclic fields of the compositum to have degree ℓ, the requirement is weakened to only be that their degrees are arbitrary powers of ℓ; assuming this, an explicit description of new circular units is given, and to complement the result, the index of the enlarged group of circular units in the full group of units of K is calculated. Finally, similarly to [2], these units are used to give a result on the annihilation of the ℓ-Sylow subgroup of the ideal class group of K.

References

[1] W. Sinnott, *On the Stickelberger ideal and the circular units of an abelian field,* Invent. Math. 62 (1980), 181–234, doi: 10.1007/BF01389158.

[2] C. Greither, R. Kučera, *On the compositum of orthogonal cyclic fields of the same odd prime degree*, Canad. J. Math. 73 (2021), 1506–1530.

Ingrid Vukusic

On some p-adic valuations and some Lebesgue integrals

Abstract: I will present two independent cute little results from my postdoc time in Waterloo. First, we will prove an explicit formula for the p-adic valuation of the Legendre polynomials $P_n(x)$ evaluated at a prime p. Then we will solve a Lebesgue integral variant of the following problem (known as the "additive square problem"): Does there exist an infinite word over a finite alphabet, such that no two consecutive blocks of the same length have the same sum? Based on joint work with Max A. Alekseyev, Tewodros Amdeberhan, and Jeffrey Shallit.

Mikuláš Zindulka

Negative bias in moments of the Legendre family of elliptic curves

Abstract: Non-holomorphic modular forms can be successfully applied to various problems about the distribution of traces of Frobenius of elliptic curves. The number of elliptic curves $E/\mathbb{F}_{p^r}$ with a fixed trace t is essentially given by the Hurwitz class number $H(4p^r - t^2)$. The generating function for the Hurwitz class numbers is in turn a mock modular form, in other words, the holomorphic part of a weight $\frac{3}{2}$ harmonic Maass form. This was the starting point for a recent result of Bringmann, Kane, and Pujahari, who showed that the traces of Frobenius in arithmetic progressions are equidistributed with respect to the Sato-Tate measure.
In this talk, I will apply these techniques to the distribution of traces in the Legendre family of elliptic curves. The influential Negative Bias Conjecture of S. J. Miller states that the bias in the second moment is always negative for every family with a non-constant j-invariant. Building on a paper by Ono, Saad, and Saikia, I will give an expression for the higher moments of the Legendre family and show that each lower order term in the asymptotic expansion is either zero or negative on average. The talk is based on a joint work with Ben Kane.