List of lectures for the *ProDoc-Workshop on p-adic periods* Alpbach/Austria, July 18-23, 2010

This workshop is organized by G. Wüstholz (Chair), A. Kresch, C. Fuchs and with the special help of L. Berger (ENS Lyon) within the ProDoc module *Arithmetic and Geometry*. It takes place from July 18-23, 2010 in Alpbach, Austria.

1. Galois representations and cohomology

1.1. **Galois representations.** Definition, examples, semilinear B-reps and elements of $H^1(G_K,\mathrm{GL}_d(B))$, see Ch 2 of [1].

1.2. **Hilbert's theorem 90.** See [15] or Ch 2 of [1].

1.3. $H^i(G_K, V)$**.** Recall their dimensions, vanishing for $i \geq 3$ and Tate duality, see [15]. No proofs.

2. Ramification of local fields

2.1. **The ramification filtration and conductors.** See [14] or [12] or Ch 3 of [1].

2.2. **The field $\mathbf{C}_p$.** $\mathbf{C}_p$ algebraically closed and complete, see Ch 3 of [1].

2.3. **Ax-Sen-Tate's theorem.** See Ch 3 of [1] or §3 of [6].

3. Witt vectors

3.1. **Teichmüller lifts.** See [14] or Ch 5 of [1].

3.2. **Witt vectors.** See [14] or Ch 5 of [1].

3.3. **Witt vectors over valued rings.** Optional - see Ch 5 of [1].

4. Cyclotomic extensions and the cohomology of $\mathbf{C}_p$

4.1. **Regularity.** Ch 8 of [1].

4.2. **Tate's traces.** Ch 8 of [1].

4.3. **Cohomology of $\mathbf{C}_p$ and $\mathbf{C}_p(i)$.** Ch 8 of [1]. The H^0 is needed but the H^1 is not as important.

4.4. **$\mathbf{C}_p$-admissible characters.** Sen's theorem: A character of G_K is $\mathbf{C}_p$-admissible iff it is of the form $\mu \cdot \alpha$ where μ is unramified and α is finite. If $K = \mathbf{Q}_p$ this follows from the above but if $K \neq \mathbf{Q}_p$ it's much harder. The proof is in [13].

5. The rings $\widetilde{\mathbf{E}}^+$ and $\widetilde{\mathbf{A}}^+$ and $\widetilde{\mathbf{B}}^+$

5.1. **Construction.** Ch 9 of [1] or §7 of [6] (note that there $\widetilde{\mathbf{E}}^+$ is denoted by $\mathcal{R}$) or [7] (same comment).

5.2. **$\widetilde{\mathbf{E}}$ algebraically closed.** Ch 9 of [1].

5.3. **The action of $G_{\mathbf{Q}_p}$ on $\widetilde{\mathbf{E}}$.** Optional - Ch 9 of [1].

5.4. $\widetilde{\mathbf{A}}^+$**,** $\widetilde{\mathbf{B}}^+$**.** Ch 9 of [1] or §7 of [6] (note that there $\widetilde{\mathbf{A}}^+$ is denoted by $\mathbf{A}_{inf}$) or [7] (same comment).

5.5. θ **on** $K \otimes_{K_0} \widetilde{\mathbf{B}}^+$**.** Ch 15 of [1] or §7 of [6].

6. The field $\mathbf{B}_{\mathrm{dR}}$

6.1. **Construction.** Ch 15 of [1] or [7].

6.2. $\mathbf{B}_{\mathrm{dR}}^{G_K}$**.** Ch 15 of [1] or [7].

6.3. **de Rham representations.** See [8].

7. The ring $\mathbf{B}_{\max}$

Fontaine constructs a ring $\mathbf{B}_{\mathrm{cris}}$ in [7] but the closely related ring $\mathbf{B}_{\max}$ has the same properties and is much better behaved.

7.1. **Construction.** See [7] or §7 of [6] or also Ch 14 of [1].

7.2. $K \otimes_{K_0} \mathbf{B}_{\max} \hookrightarrow \mathbf{B}_{\mathrm{dR}}$. See §7 of [6] or also Ch 15 of [1].

7.3. **Computation of** $\mathbf{B}_{\max}^{\varphi=\lambda} \cap \mathrm{Fil}^h \mathbf{B}_{\mathrm{dR}}$**.** See §8 of [6] or also Ch 16 of [1].

8. Formal groups

8.1. **Formal groups.** See [12].

8.2. **Differentials, logarithm.** See [12].

8.3. **Cartier dual, de Rham cohomology.** Optional.

9. Lubin-Tate modules and class field theory

9.1. **Lubin-Tate modules.** See [12].

9.2. **Tate modules of Lubin-Tate modules.** See [12].

9.3. **Class field theory.** See [12].

10. Filtered φ-modules and crystalline representations

10.1. **Filtered** φ**-modules.** See [8].

10.2. t_H **and** t_N**, admissible objects.** See [8] or also Ch 16 of [1].

10.3. **Crystalline representations.** See [8] or also Ch 16 of [1].

10.4. **Examples from geometry (no proofs).** See [8], and mention the Fontaine-Mazur conjecture(s).

11. Bloch-Kato's exponential

11.1. **The fundamental exact sequence.** See §8 of [6] or also [5] or [11].

11.2. **Bloch-Kato's exponential.** See [3] or [11].

11.3. **Interpretation for formal groups and Kummer theory.** See 3.10 of [3].

11.4. **Dimensions of the $H^1_*(K,V)$.** Optional - see [11].

12. Periods of Lubin-Tate modules

12.1. **Colmez' construction of periods.** See §8 of [6]. It is not really said but it is the case that the element t_E which he constructs is a period for T_pG where G is the corresponding Lubin-Tate module.

12.2. $\mathrm{D}_{\mathrm{cris}}(T_pG)$ **for a Lubin-Tate module.** Follows from the above, $\mathrm{D}_{\mathrm{cris}}(T_pG) = \oplus_{k=0}^{d-1} E \cdot \varphi^k(t_E)$.

12.3. **Transcendence degree of the periods.** The action of Galois should allow one to compute the tr degree of $\overline{\mathbf{Q}}_p((\varphi^k(t_E))_{k=0}^{d-1})$ inside $\mathbf{B}_{\mathrm{dR}}$.

13. The period pairing

13.1. **The pairing $T_pG \times H^1_{\mathrm{dR}}(G) \to \mathbf{B}^+_{\mathrm{dR}}$ for formal groups.** See [4].

13.2. **Elliptic curves and abelian varieties.** See [4].

Course notes and articles

There are online course notes by Berger and by Colmez as well as their articles and a book by Fontaine and Ouyang here:

- `http://www.umpa.ens-lyon.fr/~lberger/publications.html`
- `http://www.umpa.ens-lyon.fr/~lberger/ihp2010.html`
- `http://www.math.jussieu.fr/~colmez/publications.html`
- `http://www.math.jussieu.fr/~colmez/Enseignement.html`
- `http://www.math.u-psud.fr/~fontaine/recherche.html`

The numbering of § in [6] refers to the version on Colmez' webpage.

References

[1] L. Berger, Galois representations and (φ,Γ)-modules, *online notes*.
[2] L. Berger, An introduction to the theory of p-adic representations.
[3] S. Bloch and K. Kato, L-functions and Tamagawa numbers of motives.
[4] P. Colmez, Périodes p-adiques des variétés abéliennes.
[5] P. Colmez, Théorie d'Iwasawa des représentations de de Rham d'un corps local.
[6] P. Colmez, Espaces de Banach de dimension finie.
[7] J-M. Fontaine, Le corps des périodes p-adiques.
[8] J-M. Fontaine, Représentations p-adiques semi-stables.
[9] J-M. Fontaine, Arithmétique des représentations galoisiennes p-adiques.
[10] J-M. Fontaine and Y. Ouyang, Theory of p-adic Galois Representations, *online manuscript*.
[11] J. Nekovar, p-adic height pairings.
[12] J. Neukirch, Algebraic number theory.
[13] S. Sen, Lie algebras of Galois groups arising from Hodge-Tate modules.
[14] J-P. Serre, Corps locaux.
[15] J-P. Serre, Cohomologie galoisienne.
[16] F. Urfels, Fonctions L p-adiques et variétés abéliennes à multiplication complexe, *PhD thesis*.