Workshop "Arithmetic and Geometry: Ten years in Alpbach"

Hotel Böglerhof, Alpbach/Austria
June 26 - July 1, 2016

Organizers: J. Ayoub, C. Fuchs, A. Kresch, G. Wüstholz
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This, the 10th in a series of Workshops in Alpbach, will Feature minicourses
and conference talks on the Topics of previous years, covering a spectrum
of modern arithmetic geometry. Minicourses presenting a broad overview of
these Topics, delivered by top international experts, will be complemented
by conference talks highlighting recent progress.

Minicourses given by:
Peter Scholze (Bonn)
Umberto Zannier (SNS Pisa)
Shou-Wu Zhang (Princeton)

Invited Talks:
Laura Capuano (SNS Pisa)
Ziyang Gao (Paris 7)
Sergey Gorchinskiy (Steklov Inst., Moscow)
Yuri Tschinkel (Courant Inst.)
Alberto Vezzani (Paris 6)
Jing Yu (NTU Taipei)

<div align="center">**Program**</div>

All lectures take place in the seminar room at Hotel Böglerhof.

Sunday, June 26, 2016:

17:00 - 17:15:   Opening
Welcome words and organizational matters

17:15 - 18:45:   **Sergey Gorchinskiy** (Steklov Inst., Moscow)
Higher-dimensional Contou-Carrere symbol

Monday, June 27, 2016:

09:00 - 10:30:   **Peter Scholze** (Bonn)
Local Shimura varieties. part I

11:00 - 12:30:   **Shou-Wu Zhang** (Princeton)
Faltings heights and L-functions. part I

13:45 - 14:45:   **Ziyang Gao** (Paris 7)
Bi-algebraic system on the universal vectorial extension and some transcendental results

15:00 - 16:00:   **Laura Capuano** (SNS Pisa)
Unlikely Intersections in certain families of abelian varieties and the polynomial Pell equation

Tuesday, June 28, 2016:

09:00 - 10:30:   **Shou-Wu Zhang** (Princeton)
Faltings heights and L-functions. part II

11:00 - 12:30:   **Peter Scholze** (Bonn)
Local Shimura varieties. part II

13:45 - 14:45:   **Yuri Tschinkel** (Courant Inst.)
Rationality problems

15:00 - 16:00:   **Alberto Vezzani** (Paris 6)
The Monsky-Washnitzer and the overconvergent realisations

Wednesday, June 29, 2016:

09:00 - 10:30: **Peter Scholze** (Bonn)
Local Shimura varieties. part III

11:00 - 12:30: **Umberto Zannier** (SNS Pisa)
Hyperelliptic Continued Fractions and Generalized Jacobians. part I

Thursday, June 30, 2016:

09:00 - 10:30: **Umberto Zannier** (SNS Pisa)
Hyperelliptic Continued Fractions and Generalized Jacobians. part II

11:00 - 12:30: **Shou-Wu Zhang** (Princeton)
Faltings heights and L-functions. part III

13:45 - 14:45: **Jing Yu** (NTU Taipei)
Linear Forms in Multiple Zeta Values, the Positive Characteristic Case. part I

15:00 - 16:00: **Jing Yu** (NTU Taipei)
Linear Forms in Multiple Zeta Values, the Positive Characteristic Case. part II

Friday, July 1, 2016:

09:00 - 10:30: **Peter Scholze** (Bonn)
Local Shimura varieties. part IV

11:00 - 12:30: **Shou-Wu Zhang** (Princeton)
Faltings heights and L-functions. part IV

Abstracts

Peter Scholze (Bonn)
Title: *Local Shimura varieties*
Abstract: In the book of Rapoport and Zink, they constructed moduli spaces of p-divisible groups with linear-algebraic extra structure. They form a p-adic local analogue of moduli space of abelian varieties with similar extra structure, which are a special class of Shimura varieties. Rapoport and Viehmann conjectured that similarly, there are local Shimura varieties generalizing Rapoport-Zink spaces; these local Shimura varieties should be defined in a purely group-theoretical manner. We will discuss the construction of these local Shimura varieties, and the relevant background from p-adic Hodge theory.

Umberto Zannier (SNS Pisa)
Title: *Hyperelliptic Continued Fractions and Generalized Jacobians*
Abstract: For a complex polynomial $D(t)$ of even degree, one may define the continued fraction of $\sqrt{D(t)}$. This was found relevant already by Abel in 1826, and later by Chebyshev, concerning integration of (hyperelliptic) differentials; they realized that, contrary to the classical case of square roots of positive integers treated by Lagrange and Galois, there is not always pre-periodicity of the partial quotients. In this short course we shall review some classical facts on these topics and prove some new results: for instance, a correct analogue of Lagrange's theorem still exists in full generality, in that pre-periodicity of the degrees of the partial quotients always holds. We shall further study other aspects of the continued fraction, like the growth of the heights of partial quotients. This analysis is related to the geometry of (generalized) Hyperelliptic Jacobians, for which we shall recall some standard theory. Some of the proofs also use a version for algebraic groups of the Skolem-Mahler-Lech theorem, which we plan to develop.

Shou-Wu Zhang (Princeton)
Title: *Faltings heights and L-functions*
Abstract: I will explain recent work on relation between Faltings heights of some Shimura varieties and special values of L-series: including Colmez' conjecture for CM points, and work of Zhiwei Yun and Wei Zhang on moduli of Shtukas, and work of Xinyi Yuan on modular curves.

Laura Capuano (SNS Pisa)
Title: *Unlikely Intersections in certain families of abelian varieties and the polynomial Pell equation*
Abstract: Given n independent points on the Legendre family of elliptic curves of equation $Y^2 = X(X - 1)(X - c)$ with coordinates algebraic over

$\mathbb{Q}(c)$, we will see that there are at most finitely many specializations of c such that two independent relations hold between the n points on the specialized curve. This result fits in the framework of the so-called Unlikely Intersections. We will see analogues of this result in certain families of abelian varieties and in a family of split semi-abelian varieties. We will finally explain some applications of these results to the study of the solvabilty of almost-Pell equations in polynomials. This is joint work with Fabrizio Barroero.

Ziyang Gao (Paris 7)
Title: *Bi-algebraic system on the universal vectorial extension and some transcendental results*
Abstract: Let $\mathfrak{A}_g^\natural$ be the universal vector extension of the universal abelian scheme. The uniformization of $\mathfrak{A}_g^\natural$ is endowed with a "natural" structure of algebraic varieties, but the uniformizing map is transcendental. In my talk I will give the characterization of bi-algebraic subvarieties of $\mathfrak{A}_g^\natural$, i.e. a subvariety of $\mathfrak{A}_g^\natural$ such that any irreducible component of its inverse image under the uniformization is algebraic. I will then state some transcendental results related to this bi-algebraic system, including the Ax logarithmique theorem, the Ax-Lindemann theorem and the Ax-Schanuel conjecture. Their relations will also be explained.

Sergey Gorchinskiy (Steklov Inst., Moscow)
Title: *Higher-dimensional Contou-Carrere symbol*
Abstract: The talk is based on a common work with D. Osipov. Contou-Carrere symbol in dimension n is a way to construct an invertible element of an arbitrary commutative ring A using $n+1$ Laurent series of n variables over A. This symbol arises when considering families of n-dimensional varieties and chains of irreducible subvarieties on them. The higher-dimensional Contou-Carrere symbol satisfies many fundamental properties, among them, a higher-dimensional reciprocity law and a certain universal property. One has several explicit formulas for the higher-dimensional Contou-Carrere symbol. All this will be discussed in detail in the talk.

Yuri Tschinkel (Courant Inst.)
Title: *Rationality problems*

Alberto Vezzani (Paris 6)
Title: *The Monsky-Washnitzer and the overconvergent realisations*

Jing Yu (NTU Taipei)
Title: *Linear Forms in Multiple Zeta Values, the Positive Characteristic Case*

Abstract: Multiple zeta values in positive characteristic was introduced in 2004 by D.Thakur, in analogy with the classical multiple zeta values originated from Euler. We report here the progress on the transcendence aspect of the theory about theses special values. In particular the phenomenon that for any set of these values with the same weight, linear independence over the rationals forces linear independence over the algebraic closure. An analogue of a strong form of Goncharov's conjecture will be proved (after C.-Y. Chang). This explains why and how the algebraic relations among multiple zeta values can be reduced to linear relations. Our method makes use of theory of t-motives and t-modules which are positive characteristic version of motives and group varieties, with a fundamental theorem of M. Papanikolas playing the role of Grothendieck's period conjecture.