

Workshop Alpbach 2023

Hotel Böglerhof, Alpbach/Austria
June 25 - 30, 2023

Organizers: J. Ayoub, C. Fuchs, Ph. Habegger, R. Pink, S. Zerbes
Supported by: ETH Zurich, University of Zurich, University of Salzburg,
Austrian Science Fund (FWF): I4406

This, the 16th in a series of workshops in Alpbach, will feature minicourses given by world class researchers and invited talks by younger researchers, covering a spectrum of in arithmetic geometry related to Galois representations and heights. The emphasis includes not only deep theoretical developments, but also applications of a more concrete/computational nature. Minicourses presenting a broad overview of these topics, delivered by top international experts, will be complemented by invited talks highlighting recent progress.

Minicourses given by:

Kiran Kedlaya (UCSD)
Bruno Klingler (HU Berlin)
Holly Krieger (U. Cambridge)

Talks given by:

Alina Bucur (UCSD)
Yohan Brunebarbe (U. Bordeaux)
Tim Gehringer (ETH Zurich)
Rafael von Känel (Tsinghua U.)
Julia Schneider (EPFL)

Program

All lectures take place in the seminar room at Hotel Böglerhof.

Sunday, June 25, 2023:

- 17:00 - 17:15: **Opening**
Welcome words and organizational matters
- 17:15 - 18:45: **Alina Bucur** (UCSD)
Counting D_4 quartic extensions of a number field ordered by discriminant

Monday, June 26, 2023:

- 09:00 - 10:30: **Kiran Kedlaya** (UCSD)
Galois representations and Fargues-Fontaine curves. I
- 11:00 - 12:30: **Bruno Klingler** (HU Berlin)
Hodge theory, between algebraicity and transcendence. I
- 13:45 - 15:15: **Yohan Brunebarbe** (U. Bordeaux)
Counting integral points on varieties with large fundamental group

Tuesday, June 27, 2023:

- 09:00 - 10:30: **Holly Krieger** (U. Cambridge)
Heights, equidistribution, and arithmetic dynamics. I
- 11:00 - 12:30: **Kiran Kedlaya** (UCSD)
Galois representations and Fargues-Fontaine curves. II
- 13:45 - 15:15: **Tim Gehringer** (ETH Zurich)
The stable (marked) reduction of hyperelliptic curves

Wednesday, June 28, 2023:

09:00 - 10:30: **Bruno Klingler** (HU Berlin)
Hodge theory, between algebraicity and transcendence. II

11:00 - 12:30: **Holly Krieger** (U. Cambridge)
Heights, equidistribution, and arithmetic dynamics. II

13:45 - 15:15: **Holly Krieger** (U. Cambridge)
Heights, equidistribution, and arithmetic dynamics. III

Thursday, June 29, 2023:

09:00 - 10:30: **Kiran Kedlaya** (UCSD)
Galois representations and Fargues-Fontaine curves. III

11:00 - 12:30: **Julia Schneider** (EPFL)
Birational maps of Severi-Brauer surfaces, with applications to Cremona groups of higher rank

Friday, June 30, 2023:

09:00 - 10:30: **Bruno Klingler** (HU Berlin)
Hodge theory, between algebraicity and transcendence. III

11:00 - 12:30: **Rafael von Känel** (Tsinghua U.)
Integral points on coarse Hilbert moduli schemes

Abstracts

Kiran Kedlaya (UCSD)

Title: *Galois representations and Fargues-Fontaine curves*

Abstract: The Narasimhan-Seshadri theorem describes an equivalence of categories between unitary representations of the fundamental group of a compact Riemann surface and vector bundles satisfying a certain stability condition. We describe an analogue of this result relating representations of the Galois group of a suitable (infinite) extension of the p -adic field \mathbb{Q}_p with vector bundles on a “curve” introduced by Fargues and Fontaine. This ends up giving a geometric interpretation of various results in p -adic Hodge theory due to Fontaine, Colmez-Fontaine, Berger, and the speaker, among others.

Bruno Klingler (HU Berlin)

Title: *Hodge theory, between algebraicity and transcendence*

Abstract: Hodge theory, as developed by Deligne and Griffiths, is an essential tool for analyzing the geometry and arithmetic of complex algebraic varieties. It is a crucial fact that at heart, Hodge theory is NOT algebraic. On the other hand, according to both the Hodge conjecture and the Grothendieck period conjecture, this transcendence is severely constrained. This minicourse will illustrate this tension between algebraicity and transcendence by focusing on the study of Hodge loci, presenting in passing the necessary tools (functional transcendence, o-minimality, etc.).

Holly Krieger (U. Cambridge)

Title: *Heights, equidistribution, and arithmetic dynamics*

Abstract: I will survey the use of height functions in arithmetic dynamics, with a focus on applications of the quantitative equidistribution theorem of Favre and Rivera-Letelier. In particular, I will give an overview of the technique of DeMarco, Ye, and myself to provide uniform bounds on common torsion points for elliptic curves in the Legendre family. The talks will cover basic concepts in arithmetic and complex dynamics and the relationship to the arithmetic of elliptic curves, computation of non-archimedean local height functions, and degenerations of one-parameter families of local dynamical height functions. I will conclude with a discussion of recent spectacular advances in proving uniform bounds in unlikely intersections, from a dynamical perspective.

Alina Bucur (UCSD)

Title: *Counting D_4 quartic extensions of a number field ordered by discriminant*

Abstract: A guiding question in number theory, specifically in arithmetic statistics, is counting number fields of fixed degree and Galois group as their discriminants grow to infinity. We will discuss the history of this question and take a closer look at the story in the case of quartic fields. In joint work with Florea, Serrano Lopez, and Varma, we extend and make explicit the counts of extensions of an arbitrary number field that was done over the rationals by Cohen, Diaz y Diaz, and Olivier.

Yohan Brunebarbe (U. Bordeaux)

Title: *Counting integral points on varieties with large fundamental group*

Abstract: We know by Falting's theorem that any smooth projective curve of genus at least 2 defined over a number field K has only finitely many K -rational points. On the other hand, both elliptic curves and the projective line can have infinitely many rational points; but elliptic curves have "much less" rational points than the projective line (this can be made precise for example by counting points of bounded height). Answering a question asked in a recent paper of Ellenberg, Lawrence and Venkatesh, we prove a higher dimensional analogue of this: on an algebraic variety defined over a number field whose fundamental group is large in the sense of Kollár-Campana, the number of integral points with bounded height has a subpolynomial growth in the height. This is joint work with Marco Maculan.

Tim Gehringer (ETH Zurich)

Title: *The stable (marked) reduction of hyperelliptic curves*

Abstract: A fundamental tool in studying the arithmetic of an algebraic variety over a number field is the theory of stable reduction. I will start by giving a survey of this theory in the case of curves and talk about the connections to Analytification. Afterwards, I will present recent work with R. Pink, where we study the stable marked reduction of hyperelliptic curves in the case of mixed characteristic $(0, 2)$.

Rafael von Känel (Tsinghua U.)

Title: *Integral points on coarse Hilbert moduli schemes*

Abstract: I will present explicit bounds for the height and the number of integral points on coarse Hilbert moduli schemes outside the branch locus. Furthermore, I will illustrate the results with examples given by certain classical surfaces and I will explain the strategy of proof which combines the method of Faltings (Arakelov, Parsin, Szpiro) with modularity and Masser-Wüstholz isogeny estimates. This is joint work with Arno Kret.

Julia Schneider (EPFL)

Title: *Birational maps of Severi-Brauer surfaces, with applications to Cremona groups of higher rank*

Abstract: Cremona groups are groups of birational transformations of a projective space. Their structure depends on the dimension and the field. After an introduction to Cremona groups, we will first focus on birational transformations of (non-trivial) Severi-Brauer surfaces, that is, surfaces that become isomorphic to the projective plane over the algebraic closure of K . Such surfaces do not contain any K -rational point. We will prove that if such a surface contains a point of degree 6, then its group of birational transformations is not generated by elements of finite order as it admits a surjective group homomorphism to the integers. As an application, we use this result to study varieties over the complex numbers that admit a fibration where the generic fibre is a non-trivial Severi-Brauer surface. We prove that the free group over an uncountable set is the quotient of any complex Cremona group of rank at least 4. This is joint work with J r my Blanc and Egor Yasinsky.