Workshop Alpbach 2024

Hotel Böglerhof, Alpbach/Austria
June 29 - July 4, 2024

Organizers: J. Ayoub, C. Fuchs, P. Habegger, R. Pink, S. Zerbes
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This, the 17th in a series of workshops in Alpbach, will feature minicourses given by world class researchers and invited talks by younger researchers, covering a spectrum of in arithmetic geometry related to Galois representations and heights. The emphasis includes not only deep theoretical developments, but also applications of a more concrete/computational nature. Minicourses presenting a broad overview of these topics, delivered by top international experts, will be complemented by invited talks highlighting recent progress.

Minicourses given by:
Dustin Clausen (IHES)
Mahesh Kakde (IIS Bangalore)
Dimitris Koukoulopoulos (U. Montreal)

Talks given by:
Greg Baldi (IHES)
Hang Fu (U. Hannover)
Giada Grossi (Paris 13)
David Loeffler (FernUni Schweiz)
Maxim Mornev (EPFL)
Francesco Zerman (FernUni Schweiz)

Program

All lectures take place in the seminar room at Hotel Böglerhof.

Saturday, June 29, 2024:

17:00 - 17:15: Opening
Welcome words and organizational matters

17:15 - 18:45: **Dimitris Koukoulopoulos** (U. Montreal)
Metric Diophantine approximation. I

Sunday, June 30, 2024:

09:00 - 10:30: **Dimitris Koukoulopoulos** (U. Montreal)
Metric Diophantine approximation. II

13:45 - 15:15: **Dimitris Koukoulopoulos** (U. Montreal)
Metric Diophantine approximation. III

Monday, July 1, 2024:

09:00 - 10:30: **Dustin Clausen** (IHES)
Three perspectives on Deligne cohomology. I

11:00 - 12:30: **Mahesh Kakde** (IIS Bangalore)
The Gross-Stark conjecture. I

13:45 - 14:45: **Greg Baldi** (IHES)
Translation surfaces and periods

15:00 - 16:00: **Maxim Mornev** (EPFL)
Local Kummer theory for Drinfeld modules (with Richard Pink)

Tuesday, July 2, 2024:

09:00 - 10:30: **Mahesh Kakde** (IIS Bangalore)
The Gross-Stark conjecture. II

11:00 - 12:30: **Dustin Clausen** (IHES)
Three perspectives on Deligne cohomology. II

Wednesday, July 3, 2024:

09:00 - 10:30: **Dustin Clausen** (IHES)
Three perspectives on Deligne cohomology. III

11:00 - 12:30: **Mahesh Kakde** (IIS Bangalore)
The Gross-Stark conjecture. III

13:45 - 14:45: **David Loeffler** (FernUni Schweiz)
P-adic regulators for Hilbert modular surfaces

15:00 - 16:00: **Giada Grossi** (Paris 13)
P-adic Asai L-function and Asai-Flach classes

Thursday, July 4, 2024:

09:00 - 10:00: **Francesco Zerman** (FernUni Schweiz)
Big Heegner points and Iwasawa theory for Hida families

10:30 - 11:30: **Hang Fu** (Hannover)
On the dynamics of quadratic polynomials

Abstracts

Dustin Clausen (IHES)
Title: *Three perspectives on Deligne cohomology*
Abstract: Deligne cohomology is a refined cohomology theory for complex manifolds, known in number theory for its role both in Arakelov geometry and in Beilinson's conjectures on special values of L-functions. In these lectures I will describe three interrelated perspectives on Deligne cohomology. The first represents an attempt to determine for which kinds of "analytic spaces" the standard definition of Deligne cohomology makes sense and has good properties. The second and third, which describe theories only conjecturally equivalent to Deligne cohomology, represent attempts to articulate that Deligne cohomology is the analytic analog of motivic cohomology. In the second, which is joint with Peter Scholze, we define (using Efimov's theory of "continuous K-theory") a version of algebraic K-theory for analytic spaces and conjecture that it is essentially the K-theory analog of Deligne cohomology. In the third we more directly articulate a conjectural universal property for Deligne cohomology inspired by the recent "non-$\mathbb{A}^1$-invariant motivic" formalism of Annala-Iwasa in the algebraic setting. The first and second perspectives make use of the theory of analytic geometry developed in joint work with Peter Scholze; we will review the necessary aspects of that theory.

Mahesh Kakde (IIS Bangalore)
Title: *The Gross-Stark conjecture*
Abstract: In the first lecture I will review the definition of the p-adic L-function for totally real number fields. I will then state the conjecture of Gross, which is a p-adic analogue of Stark's conjecture. This conjecture can be deduced from existence of global Galois cohomology classes with prescribed local behaviour.
In the second lecture I will talk about Hilbert modular forms. In particular, we will look at Eisenstein series and linear combination of their products to construct a cusp form. This leads to congruences between Eisenstein series and cusp forms.
In the third lecture I will explain Ribet's method of using congruences between modular forms to construct Global Galois cohomology classes. We will construct the classes required to prove the Gross-Stark conjecture.

Dimitris Koukoulopoulos (U. Montreal)
Title: *Metric Diophantine approximation*
Abstract: The lectures will explain the origin of the field of metric Diophantine approximation and of the Duffin-Schaeffer conjecture. We will

then review the main ideas of its proof, as well as recent developments on stronger quantitative forms of the conjecture.

Lecture 1: Khinchin's theorem and its limitations. The Duffin-Schaeffer conjecture and Catlin's conjecture. Ergodic arguments. Hausdorff dimensions.

Lecture 2: The main ideas behind the proof of the Duffin-Schaeffer conjecture.

Lecture 3: Quantitative forms of the Duffin-Schaeffer conjecture.

Greg Baldi (IHES)

Title: *Translation surfaces and periods*

Abstract: In the moduli space of translation surfaces (i.e. pairs (X, ω) of a compact Riemann surfaces and a holomorphic one form) there are some special subvarieties known as 'orbit closures'. Even if their definition is far removed from algebraic geometry, its properties are closely related to Hodge theory. After recalling their main properties (following the work of Eskin, Filip, McMullen, Mirzakhani, Mohammadi, Möller, Wright, and many others), we will see how an 'enriched' Zilber-Pink philosophy governs the distribution of the aforementioned subvarieties, giving a new and effective approach to the finiteness results of Eskin-Filip-Wright. This is a joint work with D. Urbanik.

Hang Fu (U. Hannover)

Title: *On the dynamics of quadratic polynomials*

Abstract: Let $f_t(z) = z^2 + t$. For any $z \in \mathbb{Q}$, let S_z be the collection of $t \in \mathbb{Q}$ such that z is preperiodic for f_t. In this talk, we will discuss a uniform result regarding the size of S_z over $z \in \mathbb{Q}$. This is a joint work with Michael Stoll.

Giada Grossi (Paris 13)

Title: *P-adic Asai L-function and Asai-Flach classes*

Abstract: I will talk about joint work with D. Loeffler and S. Zerbes, in which, using higher Hida theory, we construct a p-adic measure for the Asai-motive attached to a cuspidal automorphic representation of $\mathrm{Res}_{F/\mathbb{Q}}(\mathrm{GL}_2)$, where F is a real quadratic field in which p splits. I will explain the link between (non-critical values of) this p-adic L-function and the Euler system of Asai-Flach classes and discuss, if time permits, consequences towards the Bloch Kato conjecture.

David Loeffler (FernUni Schweiz)

Title: *P-adic regulators for Hilbert modular surfaces*

Abstract: I will recall the construction of a family of classes in the motivic

cohomology of Hilbert modular surfaces, which have been studied by many authors including Kings, Ramakrishnan, and Wildeshaus. In particular, Kings' work shows that the complex regulators of these classes are related to special values of Asai L-functions. I will describe an analogous computation of the p-adic syntomic regulators of these same classes, which turn out to be related to values of a p-adic Asai L-function recently constructed by my co-authors and myself; and I will explain and how this leads to new cases of the Bloch-Kato conjecture via the method of Euler systems. This is joint work with G. Grossi and S.L. Zerbes.

Maxim Mornev (EPFL)
Title: *Local Kummer theory for Drinfeld modules (with Richard Pink)*
Abstract Let φ be a Drinfeld A-module of finite residual characteristic $\bar{\mathfrak{p}}$ over a local field K. We study the action of the inertia group of K on a modified adelic Tate module $T^{\circ}_{\mathrm{ad}}(\varphi)$ which differs from the usual adelic Tate module only at the $\bar{\mathfrak{p}}$-primary component. After replacing K by a finite extension we can assume that φ is the analytic quotient of a Drinfeld module ψ of good reduction by a lattice $M \subset K$. The image of inertia acting on $T^{\circ}_{\mathrm{ad}}(\varphi)$ is then naturally a subgroup of $\mathrm{Hom}_A(M, T^{\circ}_{\mathrm{ad}}(\psi))$.
This subgroup is described by a canonical local Kummer pairing that is the central subject of our study. In particular we give an effective formula for the image of inertia up to finite index, and obtain a necessary and sufficient condition for this image to be open. We also determine the image of the ramification filtration.
The talk is aimed at non-specialists, and all the essentials will be explained.

Francesco Zerman (FernUni Schweiz)
Title: *Big Heegner points and Iwasawa theory for Hida families*
Abstract: In the early '90s, Kolyvagin showed how to use Heegner points to build anticyclotomic Euler systems for elliptic curves, obtaining as a byproduct a proof of a particular case of the BSD conjecture. In the last 30 years, there have been many successful attempts to generalize Kolyvagin's original idea to other Galois representations. The goal of this talk is to present the state of art about the construction and the usage of the so called "big" Heegner points for the representation associated with a Hida family of modular forms. I will mainly focus on the case when the classical Heegner hypothesis on the tame conductor of the family is relaxed, presenting some results from my Ph.D thesis and some previews on an ongoing project with Luca Dall'Ava.