Workshop Alpbach 2025

Hotel Böglerhof, Alpbach/Austria
June 25 - June 30, 2025

Organizers: J. Ayoub, C. Fuchs, P. Habegger, R. Pink, S. Zerbes
Supported by: ETH Zurich, University of Zurich, University of Salzburg

This, the 18th in a series of workshops in Alpbach, will feature minicourses given by world class researchers and invited talks by younger researchers, covering a spectrum of in arithmetic geometry related to Galois representations and heights. The emphasis includes not only deep theoretical developments, but also applications of a more concrete/computational nature. Minicourses presenting a broad overview of these topics, delivered by top international experts, will be complemented by invited talks highlighting recent progress.

Minicourses given by:
Konstantin Ardakov (U. Oxford)
Matt Papanikolas (Texas AMU)
Pol van Hoften (VU Amsterdam)

Talks given by:
Giacomo Ferraro (U. Heidelberg)
Dmitrii Krekov (ETH Zurich)
Ariyan Javanpeykar (Radboud U. Nijmegen)
Arun Soor (U. Oxford)
Johannes Droschl (U. Vienna)
Marti Roset Julia (Mc Gill U.)

Program

All lectures take place in the seminar room at Hotel Böglerhof.

Wednesday, June 25, 2025:

17:00 - 17:15: Opening
Welcome words and organizational matters

17:15 - 18:45: **Konstantin Ardakov** (U. Oxford)
D-modules on rigid analytic spaces. I

Thursday, June 26, 2025:

09:00 - 10:30: **Matt Papanikolas** (Texas AMU)
Periods of Anderson t-modules. I

11:00 - 12:30: **Pol van Hoften** (VU Amsterdam)
Hecke orbits on Siegel modular varieties. I

13:45 - 14:45: **Ariyan Javanpeykar** (Radboud U. Nijmegen)
The weakly special conjecture contradicts Orbifold Mordell (and hence abc)

15:00 - 16:00: **Johannes Droschl** (U. Vienna)
On the Spectrum of the Symplectic Grassmannian and the Space of Matrices

Friday, June 27, 2025:

09:00 - 10:30: **Konstantin Ardakov** (U. Oxford)
D-modules on rigid analytic spaces. II

11:00 - 12:30: **Matt Papanikolas** (Texas AMU)
Periods of Anderson t-modules. II

13:45 - 14:45: **Giacomo Ferraro** (U. Heidelberg)
The kernel of the adjoint exponential in Anderson t-modules

15:00 - 16:00: **Dmitrii Krekov** (ETH Zurich)
On the factorisation of p-adic Asai L-function associated to a base-change quadratic Hilbert modular form

Saturday, June 28, 2025:

09:00 - 10:30: **Pol van Hoften** (VU Amsterdam)
Hecke orbits on Siegel modular varieties. II

11:00 - 12:30: **Konstantin Ardakov** (U. Oxford)
D-modules on rigid analytic spaces. III

Sunday, June 29, 2025:

09:00 - 10:30: **Matt Papanikolas** (Texas AMU)
Periods of Anderson t-modules. III

11:00 - 12:30: **Pol van Hoften** (VU Amsterdam)
Hecke orbits on Siegel modular varieties. III

Monday, June 30, 2025:

09:00 - 10:00: **Marti Roset Julia** (Mc Gill U.)
Rigid cocycles for $SL(n)$ and their values at special points

10:30 - 11:30: **Arun Soor** (U. Oxford)
Relation between D-cap modules and quasi-coherent sheaves on the de Rham space

Abstracts

Konstantin Ardakov (U. Oxford)
Title: *D-modules on rigid analytic spaces*
Abstract: A large part of the minicourse will be concerned with the basic properties of algebraic D-modules on smooth complex algebraic varieties and complex manifolds. We will discuss the characteristic variety, Bernstein's inequality and holonomic D-modules. We will look at D-module operations, Kashiwara's Theorem, and Bernstein's Theorems about the preservation of holonomicity under the D-module operations. We will then move on to twisted differential operators on flag varieties and the Beilinson-Bernstein Localisation Theorem. Finally, after discussing appropriate motivation from the theory of locally analytic representations of p-adic Lie groups, we will discuss rigid analytic spaces and the theory of (equivariant) D-cap modules on these spaces.

Matt Papanikolas (Texas AMU)
Title: *Periods of Anderson t-modules*
Abstract: Drinfeld modules and, in higher dimensions, Anderson t-modules, were developed in the 1970's and 1980's as counterparts of elliptic curves and abelian varieties within the framework of function fields in positive characteristic. Building on Drinfeld's constructions of elliptic modules, Anderson, Gekeler, Goss, Hayes, Pink, Thakur, Yu, and many others established the foundations of a characteristic p theory that, while intrinsically separate, parallels well-known results and objects over number fields. Elements of this theory include function field analogues of elliptic functions, periods, Galois representations, L-functions, modular forms, Hodge structures, transcendence applications, and more.
In this lecture series we will explore various aspects of the theory of t-modules, placing special emphasis on their periods and analytic theory. The first lecture will provide an overview of the essential properties of Drinfeld modules and Anderson t-modules, including their exponential and logarithm functions along with their corresponding periods. The second lecture will focus on the theory of Anderson t-motives and the ways they can be used to determine periods of t-modules through rigid analytic functions satisfying Frobenius difference equations. In the third lecture we will investigate applications to transcendence and special values of Goss L-series.

Pol van Hoften (VU Amsterdam)
Title: *Hecke orbits on Siegel modular varieties*
Abstract: This course is about the mod p geometry of the moduli space of abelian varieties, and more specifically about the Hecke orbit conjecture.

I will first give an introduction to the basic objects of the conjecture such as Newton strata, central leaves and Igusa varieties. I will then describe a non-abelian version of Serre-Tate coordinates on the formal neighborhoods of central leaves through the lense of Dieudonne–Lie algebras. Finally, I will give an overview of the proof, specifically focusing on the role played by p-adic monodromy groups. This is based on joint work with Marco D'Addezio.

Ariyan Javanpeykar (Radboud U. Nijmegen)
Title: *The weakly special conjecture contradicts Orbifold Mordell (and hence abc)*
Abstract: Which varieties over a number field have a potentially dense set of rational points? Lang conjectured that varieties of general type over a number field have very few rational points. In 2000, guided by Lang's conjecture and in search of a converse statement, Abramovich, Colliot-Thélène, Harris, and Tschinkel formulated the "Weakly Special Conjecture": every weakly special variety over a number field has a potentially dense set of rational points. In this talk we will see how this conjecture contradicts the abc conjecture, and more precisely Campana's "Orbifold Mordell" conjecture. This is joint work with Finn Bartsch, Frederic Campana, and Olivier Wittenberg.

Giacomo Ferraro (U. Heidelberg)
Title: *The kernel of the adjoint exponential in Anderson t-modules*
Abstract: Given an algebraically closed complete valued field K over $\mathbb{F}_q$, an Anderson t-module of dimension d is given by the topological $\mathbb{F}_q$-vector space K^d, endowed with an $\mathbb{F}_q$-linear action $\phi_t = \sum_{i\geq 0} T_i\tau^i \in M_{d\times d}(K)[\tau]$, where $\tau : K^d \to K^d$ sends $(v_1,\dots,v_d)$ to $(v_1^q,\dots,v_d^q)$. In analogy with complex abelian varieties, there is an analytic map $\exp = \sum_{i\geq 0} E_i\tau^i : K^d \to K^d$—which is not necessarily surjective—such that $\phi_t \exp = \exp T_0$.
The adjoint exponential, defined as the series $\exp^* := \sum_{i\geq 0} \tau^{-i}E_i^T$, determines a (non-analytic) continuous map $K^d \to K^d$. Using the factorization properties of $K[\![x]\!]$, Poonen proved that there is a perfect duality of topological $\mathbb{F}_q$-vector spaces $\ker(\exp) \times \ker(\exp^*) \to \mathbb{F}_q$ under the condition $d = 1$. In this talk, I explain that for an arbitrary *abelian* Anderson t-module, we have a collection of perfect pairings $\ker(\phi_{t^n})\times\ker(\phi^*_{t^n}) \to \mathbb{F}_q$, and that we can use them to obtain a canonical generating series $(F_\phi)_c \in M_{d\times d}(K)[\![\tau^{-1},\tau]\!]$ for all $c \in \mathbb{F}_q(\!(t^{-1})\!)/\mathbb{F}_q(t)$. The study of the properties of F_ϕ allows us to prove that, if exp is surjective, $\ker(\exp^*)$ is compact and isomorphic to the Pontryagin dual of $\ker(\exp)$. Moreover, we deduce an alternative explicit description of the Hartl-Juschka pairing, obtained by Gazda and Maurischat in a recent preprint.

Dmitrii Krekov (ETH Zurich)
Title: *On the factorisation of p-adic Asai L-function associated to a base-change quadratic Hilbert modular form*

Abstract: Let $\mathcal{F}$ be a quadratic Hilbert modular form which is a base-change of a classical new p-ordinary non-CM eigenform f. One can associate to $\mathcal{F}$ a certain Galois representation called the Asai representation which turns out to be explicitly decomposable as a direct sum of two representations. Therefore one obtains a corresponding decomposition of the complex L-function associated to the Asai representation. Recently G. Grossi, D. Loeffler and S. Zerbes defined p-adic (imprimitive) Asai L-function so it makes sense to ask whether the factorisation above has a p-adic analogue. This question turns out to be quite non-trivial due to the abscence of critical values for the parallel-weight Asai L-function. However, the problem can be tackled by adapting a method used by S. Dasgupta in a similar problem of factorisation of p-adic L-function associated to the Rankin-Selberg self-convolution of a modular form. I will discuss some recent progress in this direction as well as potential applications if time permits.

Arun Soor (U. Oxford)
Title: *Relation between D-cap modules and quasi-coherent sheaves on the de Rham space*
Abstract: In algebraic geometry, there is an equivalence between the category of D-modules (on a smooth scheme X) and the category of quasi-coherent sheaves on a certain stack X_{dR} associated to X.
I will explain an analogous relation in rigid-analytic geometry. Namely, I will explain how to produce a fully-faithful functor from the category of coadmissible D-cap modules of Ardakov-Wadsley (more generally, the C-complexes of Bode) to the category of quasi-coherent sheaves on the"analytic de Rham space".

Johannes Droschl (U. Vienna)
Title: *On the Spectrum of the Symplectic Grassmannian and the Space of Matrices*
Abstract: Spherical varieties have long played a prominent role in the Langlands correspondence, particularly through their connection to certain period integrals. In recent years, many of these phenomena have been organized under what is now known as the relative Langlands program.
In this talk, we will focus on cases closely related to the local theta correspondence and explain how local structure theorems and so-called ρ-derivatives allow us to compute their spectra.

Marti Roset Julia (Mc Gill U.)
Title: *Rigid cocycles for SL(n) and their values at special points*
Abstract: The theory of complex multiplication implies that the values of modular functions at CM points belong to abelian extensions of imaginary quadratic fields. In this talk, we propose a conjectural extension of this phenomenon to the setting of totally real fields. Generalizing the work of Darmon, Pozzi, and Vonk, we construct rigid cocycles for SL(n), which play the role of modular functions, and define their values at points associated with totally real fields. The construction of these cocycles originates from a topological source: the Eisenstein class of a torus bundle. This is ongoing joint work with Peter Xu.