ETH

Eidgenössische Technische Hochschule Zürich
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Number Theory Seminar

Multiplicative dependence on algebraic varieties

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ABSTRACT: Let $\mathcal{X}$ be an algebraic variety in affine space $\mathbf{A}^n$. We are interested in multiplicative relations $x_1^{a_1} \cdots x_n^{a_n} = 1$ holding on points of $\mathcal{X}$; but here the integer exponents are not considered fixed. When r is the dimension of $\mathcal{X}$, it is not difficult to see that there are infinitely many points for which there are r independent relations. Several people have conjectured what the effect of one more relation might be. We briefly review what is known for curves $(r = 1)$ and surfaces $(r = 2)$. At the other end of the scale, everything is classical for hypersurfaces $(r = n - 1)$. We describe some of the ideas behind recent work of Bombieri, Zannier and the speaker which settles the codimension two case $(r = n - 2)$ when $\mathcal{X}$ is defined over the field of algebraic numbers. A very simple example: there are at most finitely many complex x, y for which there are three independent multiplicative relations between the non-zero numbers x, y, $x + y + 2$, $x - y + 3$.

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