Exact Testing and exact distributions by testing

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Many statistical problems defined in applications end up with a complicated hypothesis testing. Such statistics may have complicated, unknown or untractable exact, even asymptotic distributions. For example, consider asymptotical behavior of likelihood ratio test statistic at singular points of hypothesis (see [Drton (2009)]).

However, many times, exact distribution of the statistics may be derived. We will discuss some methods of derivation of exact or nearly exact test. For instance, we will derive exact likelihood ratio test of simple hypothesis for scale parameter in the case of Type I and progressively Type II censored Weibull samples (see [Balakrishnan and Stehlík (2008)]) and procedure for simulation of quantiles for homogeneity tests in the case of Weibull with subpopulation model (see [Stehlík and Wagner (2011)]). We will illustrate such a tests to be comparable with others, and in some setups even superior to frequently used tests for exponential homogeneity which are based on the EM algorithm (like the MLRT, the ADDS test, and the D-tests). One important example of such superiority is the case of lower contamination.

In reliability engineering, the inference problem for the complete samples and large data sets are commonly rare events. Typically, missing data are present and censoring has been applied. Moreover, samples are frequently small because of many reasons (e.g. expensive observations or rare event structure of the failure process). During the talk we will discuss recent results on the exact likelihood ratio tests of scale and homogeneity hypotheses when samples are from exponential, Erlang, gamma (see [Stehlík (2003)]), Weibull (see [Stehlík (2006)]) and generalized gamma distributions (see [Stehlík (2008)]). The asymptotical tests are typically oversized and thus inappropriate for small samples. We will mention the reliability prediction when some data is missing or is censored. The reliability prediction when some data is missing plays a major role in many reliability programs (e.g. for a variety of reasons over 90% of the data in the Reliability Analysis Center does not have the individual failure times recorded, see [Coit and Jin (2000)]). We will provide also recent results for exact testing with missing data (see [Stehlík (2008)]). The real data examples and various applications (see [Stehlík and Ososkov (2003)]) will illustrate the topics discussed.

References

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